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PRINCETON UNIVERSITY

DEPARTMENT OF AERONAUTICAL ENGINEERING

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R E S T R I C T E D

DEPARTMENT OF THE NAVY
OFFICE OF NAVAL RESEARCH
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ON THE DESIGN OF MODELS

FOR

HELICOPTER RESEARCH AND DEVELOPMENT

Aeronautical Engineering Laboratory

Report No. 240

October, 1953

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A. A. NIKOLSKY

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R E S T R I C T E D

CONTENTS

Summary

An attempt is made to present some of the problems, usefulness and limitations of dynamic model testing as it pertains to the helicopter. Reduction factors for the design of dynamically similar models are presented. It is shown that a model designed as specified would be expected to exhibit stability and control characteristics similar to the prototype as well as similar rotor blade vibrations, deflections and flutter characteristics.

1. Introduction

Flight tests of a small-scale helicopter model conducted at this establishment (Ref. 1) have yielded a great deal of information on the nature of the stability and control problem. The model used was a general research vehicle and no attempt was made to simulate a specific prototype helicopter. The success of these tests, however, has aroused interest in the possibility of direct model simulation of prototype helicopter dynamic characteristics. Therefore an attempt is made herein to discuss some of the problems, usefulness and limitations of model testing as it pertains to the helicopter.

Model testing is usually resorted to for the following purposes:

(1) A model may be built to serve as a test case for the development of an analytical theory.

(2) A physically similar model may be built to obtain the performance of the prototype without resorting to the development or use of an analytical theory.

The first case is naturally much more general and is not directly concerned with a particular prototype. The model is simpler to design than in the second case since, as will be shown, there are less restrictions on its design. For research this first approach is usually sufficient. However, for engineering development, which usually cannot wait until all the phenomena involved are satisfactorily understood, the second approach is utilized. It is this approach, the design of physically similar models, that is of immediate concern. The principles, however, are equally applicable to both types of models.

In most instances it has been found to be more feasible to conduct tests using small-scale replicas since greater control of the test conditions can be exercised, and since construction and test of the model and any modifications are

relatively inexpensive. However, it must not be assumed that model studies provide ready answers to all questions. In fact, it may be stated that unless the general aspects of the phenomenon that is to be investigated are understood, a suitable model test cannot be devised nor can the results of the test be interpreted. The design of a physically similar model represents a good sized engineering job. Time and money are wasted by a test of a model that does not adequately represent the prototype.

In general, the analyses and conclusions presented herein are not new. However, with an expected increase in the use of models for rotary wing investigations a need existed to collect and summarize existing knowledge as it relates to this field.

In the following section the general principles of model design are treated. In the sections which follow the application of the theory to the problems of stability and control and rotor blade vibrations, deflections and flutter are discussed.

2. General Principles

To gain an understanding of the problems, usefulness, and limitations of model testing a discussion of the "Pi theorem" (Ref. 2) is appropriate. Only a brief discussion is presented herein in order to bring the essential points to mind. For greater detail the use of either Ref. 2 or 3 is suggested.

The behavior of a given physical system is, in general, dependent upon certain specific parameters, for example, the length, mass, velocity, and acceleration of the system; the gravitational attraction; the applied force on the system; etc. These parameters are expressed in terms of "m" fundamental units, which for a mechanical system are length, mass and time, (thus m = 3). For a thermodynamical system a fourth fundamental unit, temperature, would be considered.

Bridgman's "Pi theorem" states that the behavior of a given system which is dependent upon say "n" parameters can be described by the relation,

$$F(\pi_1, \pi_2, \pi_3, \ldots\ldots \pi_{n-m}) = 0$$

where the Pis are (n - m) dimensionless products, derived by grouping the n parameters. For example, in fluid dynamics a few of the well known Pis are Reynold's No., Pressure coefficient, and Mach No. The detailed method of grouping is presented in Refs. 2 and 3 and therefore will not be repeated herein.

The solution in the form above may be solved explicitly for any one of the products, giving an equivalent form of the result,

$$\alpha = \beta^{x_1} \gamma^{x_2} \phi(\pi_2, \pi_3, \ldots\ldots \pi_{n-m})$$

where α, β, and γ are a few of the parameters involved in the system and the x's are such that ($\alpha\beta^{-x_1}\gamma^{-x_2}$) is dimensionless (i.e. $\frac{\alpha}{\beta^{x_1}\gamma^{x_2}} = \pi_1$).

It can readily be seen that in passing from one physical system to another, the arbitrary function, ϕ, will in general change in an unknown way, so that little if any useful information could be obtained by indiscriminate model experiments. However, if the models are chosen in such a restricted way that all the Pis of the unknown function have the same value for the model as for the full-scale example, then the only variable in passing from the model to full-scale is in the factors outside of the functional sign, and the manner of variation of these factors is known. Thus, two systems which are so related to each other that the arguments, (Pis) inside the unknown functional sign are numerically equal are considered physically similar systems.

It is evident that a model experiment can give valuable information if the model is so constructed and tested that it is physically similar to the full-scale example. It should be noted that the condition of physical similarity involves, in general, not only conditions on the dimensions of the model but on all other physical variables as well.

3. Applications

A. Stability and Control

The parameters which enter into the problem of helicopter stability and control are:

1)	ρ	= mass density of air	slugs/ft.^{3}
2)	μ	= viscosity of air	slugs/ft-sec
3)	g	= gravitational acceleration	ft/sec^2
4)	T	= rotor thrust = helicopter weight	slugs ft/sec^2
5)	Ω	= rotor angular velocity	per sec.
6)	R	= rotor blade length	ft.
7)	c	= rotor blade chord	ft.
8)	V	= forward flight velocity	ft./sec.
9)	θ	= angles, (i.e. blade or fuselage angle of attack)	rad.
10)	I	= moment of inertias, (i.e. fuselage or rotor blade)	slug-ft.^{2}
11)	h	= distance, (i.e. height of rotor hub above center of gravity)	ft.
12)	b	= no. of blades	

Other parameters may be listed such as the mass of the helicopter, but this parameter is not independent since it is accounted for by the helicopter weight and gravity (T/g).

Applying the principles of Section 2, it is seen that since n = 12 and m = 3, there should exist nine dimensionless products. These products are presented below. By substituting, for example, A (blade area) for R^2 and inserting constants, which obviously does not change the character of the dimensionless product, their familiar form can be obtained. In the following,

a = airfoil slope of the lift curve, and I_1 = blade moment of inertia about its flapping hinge, the other symbols having been previously defined.

Pi	Familar form	
$\Pi_1 = \frac{\rho \Omega R c}{\mu}$	$\frac{\rho(.75\Omega R)c}{\mu}$	Reynold's No. at 3/4 radius.
$\Pi_2 = \frac{V^2}{gR}$	$\frac{V^2}{gR}$	Froude No.
$\Pi_3 = \frac{T}{\rho R^2 (\Omega R)^2}$	$\frac{T}{\rho A (\Omega R)^2}$	Thrust coefficient
$\Pi_4 = \frac{V}{\Omega R}$	$\frac{V}{\Omega R}$	Advance Ratio
$\Pi_5 = \frac{c}{R}$	$\frac{bc}{\pi R}$	Blade Stability
$\Pi_6 = \frac{h}{R}$	$\frac{h}{R}$	
$\Pi_7 = \theta$	θ	
$\Pi_8 = \frac{\rho c R^4}{I}$	$\frac{\rho a c R^4}{I_1}$	Blade Lock number
$\Pi_9 = b$	b	Number of blades

With all the above Pi s to be simulated it is expected that the design of a model would involve compromises between the conditions desired and those obtainable. For instance, consider the Pi s which involve blade rotational velocity (Ω), Π_1 , Π_3 and Π_4. It would be difficult to satisfy all these conditions for velocity since there are basically only three arbitrary reduction factors (mass, length, and time) and one, the length is not really

arbitrary because it is usually desirable to make the model smaller than full scale or it is hardly a model. In addition to conditions that are contradictory by their nature when there are not enough arbitrary choices, there are conditions fixed by the system in which it will be necessary to operate the model. For instance for tests conducted in the atmosphere the earth's gravitational attraction (g), air viscosity (μ) and density (ρ) are fixed. Under these conditions it is noticed that if Reynold's No. (π_1) is satisfied, an attempt to satisfy the thrust coefficient (π_3) would require the mass (or thrust) of the model to equal the mass (or thrust) of the prototype which obviously does not yield a "model."

Having determined the appropriate Pi s and recognizing the difficulties encountered in simulation of these dimensionless products recourse must now be taken to study the effects of variations on the results. The necessity for simulation of π_2 and π_3 could have been foreseen by a general analysis of the phenomenon. In stability and control maneuvers, three classes of forces are activated, namely,

(1) aerodynamic

(2) gravitational

(3) inertial

Froude No. (π_2) represents the ratio between inertial and gravitational forces. The thrust coefficient (π_3) represents the ratio between gravitational and aerodynamic forces. Thus, it is to be expected that for similarity of helicopter motions, these three classes of forces must be kept in proper proportion, which is guaranteed by preserving the full-scale value of π_2 and π_3.

Reynold's number enters the problem solely due to its effect on the aerodynamic forces, it being a ratio of the aerodynamic forces due to dynamic and viscous fluid actions. Thus to insure that the aerodynamic forces in themselves are simulated, the effect of Reynold's number must be considered. However, it must be kept in mind that for stability and control similarity, complete aerodynamic similarity (i.e., use of the same airfoil type) is not required, but rather the aerodynamic characteristics must be simulated.

The above relationships are pictured on the next page in Fig. 1.

If the necessity of simulating R.N. (Π_1) is relaxed, all other products may be simulated by use of the reduction factors presented in Table 1 (pg.11), in which

$$\lambda^{-1} = \text{scale factor} = \frac{\text{linear dimension of model}}{\text{linear dimension of prototype}}$$

Thus if a fifth scale model of the prototype is to be built, $\lambda^{-1} = 1/5$.

Stability and Control

Three classes of forces activated

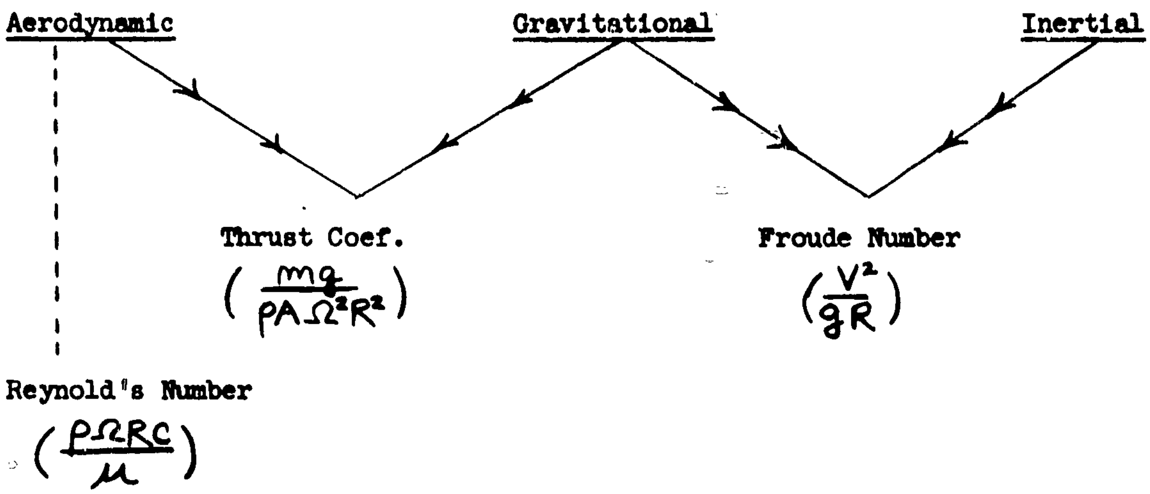


FIG. 1

The relationship between the three classes of forces activated in stability and control manoeuvers.

Table 1

Reduction Factors

Parameter	Factor
Linear Dimensions	λ^{-1}
Area	λ^{-2}
Volume, Mass, Force	λ^{-3}
Moment	λ^{-4}
Moment of Inertia	λ^{-5}
Linear Velocity	$\lambda^{-\frac{1}{2}}$
Linear Acceleration	1
Angular Velocity	$\lambda^{\frac{1}{2}}$
Angular Acceleration	λ
Time	$\lambda^{-\frac{1}{2}}$
R.P.M.	$\lambda^{\frac{1}{2}}$
Disc Loading (T/A)	λ^{-1}
Power Loading	$\lambda^{\frac{1}{2}}$
Power	$\lambda^{-3.5}$
R. N.	$\lambda^{-1.5}$
Mach No.	$\lambda^{-\frac{1}{2}}$

It is apparent that while the mechanical aspects of simulation can be satisfied in a rigorous manner, the aerodynamic aspects are relatively difficult. As shown in Table 1 (pg. 11), R. N. is reduced as $\lambda^{-1.5}$ and Mach No. by $\lambda^{-\frac{1}{2}}$. Consequently large differences between the full-scale and model Reynold's and Mach numbers are to be expected. This condition obviously would alter the aerodynamic characteristics of a given airfoil or fuselage shape (Sect. 4).

Specifically, the aerodynamic characteristics which play a significant role in stability and control manoeuvers are airfoil slope of the lift curve, drag polar and maximum lift coefficient as well as fuselage drag and moment coefficients. It is beyond expectation to quantitatively simulate all of the above parameters at the same time. However, certain prototype airfoil characteristics may be simulated at the model R. N. by change in airfoil type. For example, the increased drag at low R. N. may be offset somewhat by use of thinner sections, maximum lift coefficient (for high forward speed condition) may be retained by use of slotted airfoils or other devices utilized by the aerodynamicist to modify airfoil characteristics. Airfoil data at low R. N. is scarce, therefore a certain amount of wind tunnel and thrust stand testing will be required to evaluate the aerodynamic discrepancies between prototype and model characteristics. Attemps may then be made to minimize large discrepancies through modifications. However, caution must be exercised in modifying, for example, fuselage shape since all the effects of low R. N. may not be circumvented and still others may be introduced. This is readily apparent when the complex flow about the fuselage due to rotor downwash is considered. For conditions that cannot be simulated the only recourse is to determine either experimentally or analytically the effect of the variation on

the results. Thus it can be seen that the design and test of an appropriate helicopter model requires a large degree of skill and knowledge.

Having determined the scale factors the question naturally arises as to whether or not it is mechanically feasible to build such a model. Both single-rotor and tandem-rotor helicopters have been studied at this establishment and it appears that these scale factors result in models that can be built to be self powered and remotely controlled. The prototype and model parameters for both a typical single and tandem-rotor helicopter are presented in Tables 2 and 3 (pgs. 14 and 15). Of course, numerous design problems must be overcome, however, as stated, preliminary designs completed at this establishment appear very promising. A report covering the design of a model of the HUP tandem helicopter is soon to be released (Ref. 4). For a more detailed discussion of the design problems that must be faced, use of this reference is suggested.

B. Rotor Blade Vibrations

Having determined the scale factors for simulation of the stability and control characteristics (Sec. 3A), the question naturally arises as to whether such a model could also simulate the high frequency vibrations such as usually exhibited by the rotor blades. Consideration of these vibrations introduces, in addition to the three classes of forces mentioned in Sect. 3A, a fourth namely the structural stiffness of the structure.

The non-rotating natural frequency, ω_s, of a uniform blade may be written as,

$$\omega_s = a_n\sqrt{\frac{EI}{\mu R^4}}$$

where EI is the bending stiffness of a section, R is the length of the blade, μ is the mass per unit length, and a_n is a constant depending upon the mode of

Table 2

Typical 5000 lb. Single Rotor Helicopter

$\lambda^{-1} = 1/5$

	Prototype	Model
Gross weight, lbs.	5000	40
Blade radius, ft.	24	4.8
Rotor angular velocity, rad./sec.	20	44.7
Blade tip speed, ft./sec.	480	215
Blade chord, ft.	1.5	0.3
Number of blades	3	3
Disk loading	2.76	0.55
R. N. at 3/4 radius	3.45×10^6	3.09×10^5
Power to hover	296 HP	1.1 HP

Table 3

HUP-2 Tandem Helicopter

$$\lambda^{-1} = \frac{1}{5.85}$$

	Prototype	Model
Gross weight, lbs.	5700	28.5
Pitching moment of inertia, slug ft.^{2}	10714	1.55
Number of blades	3 (metal)	3
Blade radius, ft.	17.58	3
Blade chord (constnat) in.	13	2.22
Rotor angular velocity, rad./sec.	30.6	73.8
Spanwise flapping hinge offset, in.	2	0.341
Chordwise feathering axis offset, in.	1 fwd.	0.170
Blade flapping moment of inertia, slug ft.^{2}	163.94	0.0239
Blade static moment, slug ft.	15.06	0.0128
R. N. at 3/4 radius	2.8×10^6	1.95×10^5
Distance between rotors, in.	263	44.8
Distance from c.g. to fwd. rotor, in.	131.5	22.4
Height of fwd. rotor above c.g., in.	64.1	10.94
Height of rear rotor above c.g., in.	93.1	15.9

vibration. The dimensionless blade natural frequencies during rotation may be expressed as,

$$\left(\frac{\omega_r}{\Omega}\right)^2 = \left(\frac{\omega_s}{\Omega}\right)^2 + K$$

where ω_r is the blade vibrational frequency, Ω is the blade rotational velocity and K is a constant depending upon the blade vibrational mode shape. Thus for simulation,

$$\left(\frac{\omega_s}{\Omega}\right)_m = \left(\frac{\omega_s}{\Omega}\right)_p$$

or

$$\left(\frac{1}{\Omega}\sqrt{\frac{EI}{\mu R^4}}\right)_m = \left(\frac{1}{\Omega}\sqrt{\frac{EI}{\mu R^4}}\right)_p$$

where the subscripts m and p refer to the model and prototype respectively.

Using the same scale factors as presented in Table 1 (pg. 11), there results that for simulation,

$$\frac{(EI)_m}{(EI)_p} = \lambda^{-5}$$

Similarly it can be deduced that the torsional stiffness (GJ) of the blade must be reduced so that,

$$\frac{(GJ)_m}{(GJ)_p} = \lambda^{-5}$$

While the above discussion is based on a uniform blade, the generalization and scale factors hold equally for a non-uniform blade. However in this case the scale factors must be applied to each section of the blade.

It is interesting to note that the model stiffness is reduced to the fifth power of the scale factor rather than as the fourth power of the scale factor which would be the case if the model were made of the same material as

the prototype and the internal structure of the prototype was geometrically scaled throughout. Thus, the elastic modulus of the material used for constructing the model must be less than that used for constructing the prototype.

Satisfying the above structural stiffness scale factors would naturally result in the model exhibiting the same blade vibrational characteristics as the full-scale prototype. The construction of a blade with the above characteristics is discussed in Ref. (4). It is noteworthy to mention here that a blade demonstrating the above characteristics has been built at this establishment.

C. Rotor Blade Deflections

The structural deflections of a blade are proportional for the first power of the forces acting, the third power of blade length and inversely proportional to the blade stiffness. Thus, reducing the prototype by the scale factors as derived in Sect. 3A and B results in the deflections being reduced by λ^{-1}. In other words, as desired, the resulting blade deflections are reduced in the same proportion as its length.

D. Rotor Blade Flutter

While this work was underway, a paper which adequately covers this phase of model simulation was presented by Brooks (Ref. 5). The approach is somewhat different than this authors, however, the results are the same and therefore will not be duplicated herein. It is sufficient to state that the model designed in accordance with Sect. 3A and 3B will also simulate the flutter characteristics of the prototype. Here again the variation of Reynolds No. and Mach No. may alter the results (i.e. the speeds at which flutter actually occurs, particularly with reference to stall flutter). For further details the use of Ref. 5 is suggested.

4. Effect of Variations of Reynold and Mach Numbers on the Aerodynamic Forces

The principal obstacle in the way of dynamic simulation by use of small-scale dynamic models lies in the inability to duplicate the full-scale Reynolds and Mach No. This is due to the fact that the aerodynamic characteristics of the wings and fuselage vary with the Reynold's number and Mach No. To fully appreciate the problem a brief discussion of the variation of aerodynamic characteristics with these numbers is in order. The discussion as presented in Refs. 6 and 7 is ideal for this purpose.

Reynolds Number

a) Effect on Drag

The basic effects on body drag of increasing Reynolds number are:

(1) If the region is at a constant static pressure, the transition from laminar to turbulent boundary-layer flow is moved towards the leading edge. In a region of varying pressure such as over a wing the transition moves forward until it reaches the minimum pressure point.

(2) The drag coefficient of both laminar and turbulent boundary layers decreases.

The effects of these phenomena can be examined in the light of what happens to the minimum drag coefficient of a wing as the Reynolds number varies widely. The minimum drag of NACA symmetrical sections is shown on the next page in Fig. 2, and the 0012 will be taken as an example. The explanation for the variation of drag with R. N. may be advanced as follows: It is well known that a laminar boundary layer will separate from the surface of an object much more readily than will a turbulent boundary layer. Hence at a Reynolds No. of 300,000 the downstream position of the transition point permits a large amount

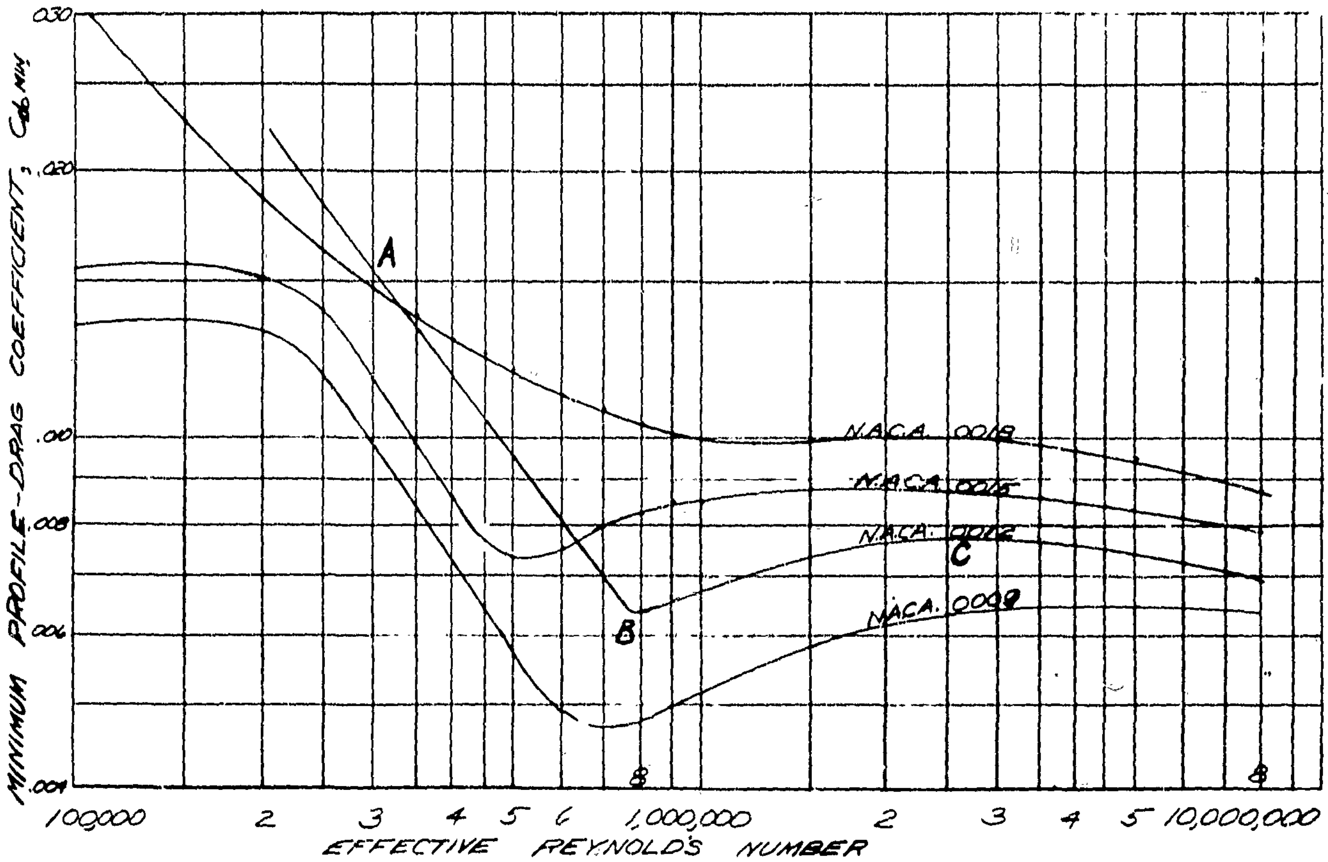
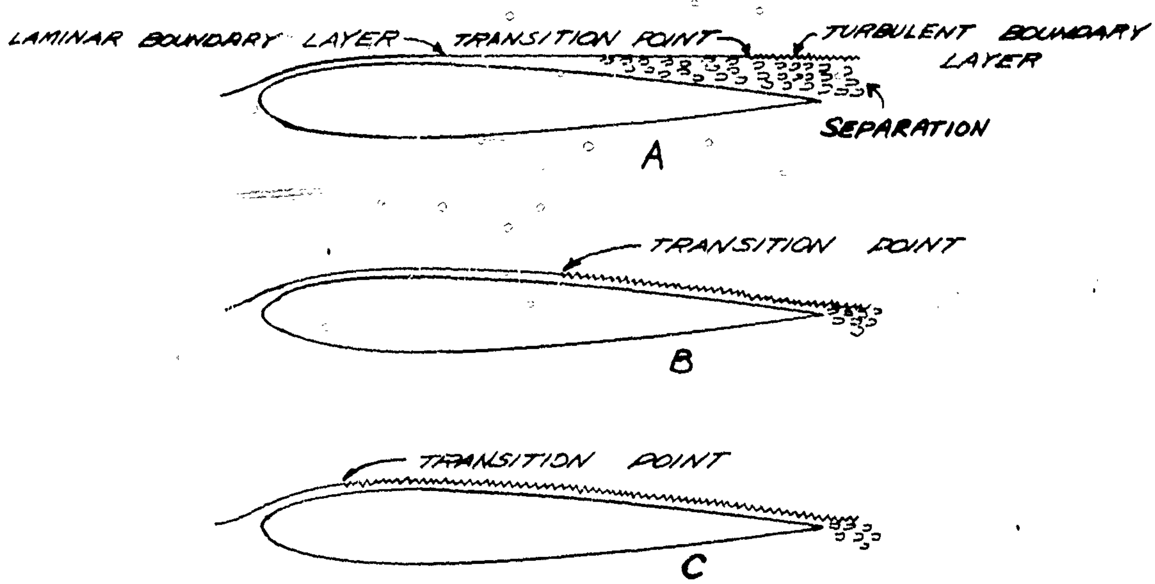


FIG. 2



EFFECT OF INCREASING REYNOLD'S NUMBER ON BOUNDARY LAYER FLOW

FIG. 3

of separation and hence large form drag. This condition corresponds to A of Figs. (2) and (3).

With increasing Reynolds number the transition region moves forward until it occurs at a point beyond which the turbulent flow remains on the wing. This flow, with the lower drag laminar flow extending a maximum along the chord, results in a low drag coefficient as shown by B in Figs. (2) and (3).

From then on, increasing the Reynolds number moves the transition region forward until it reaches the minimum pressure point. As the percentage of laminar flow decreases, the drag coefficient increases as shown from B to C in Figs. (2) and (3).

For still greater Reynolds numbers, no change in the flow pattern is supposed, the drag coefficient following the downward trend of laminar and turbulent boundary-layer flows as stated by the theory, i.e.

$$C_{D\,\text{laminar}} = \frac{2.66}{\sqrt{R.N.}}$$

$$C_{D\,\text{turbulent}} = \frac{0.91}{(\log_{10} R.N.)^{2.58}}$$

It should be noted that unlike fixed-wing aircraft, the lifting surfaces of a rotary-wing aircraft operate in their own turbulent wake. It is therefore doubtful that a large portion of the boundary layer could be laminar (i.e. of the type depicted by A in Fig. 3) since the turbulent operating conditions would induce a more rapid transition to a turbulent boundary layer. As a turbulent boundary layer adheres much more readily to the surface, the large drag due to form drag (A of Fig. 2) would not be expected to materialize. Thus the problem of simulating the drag coefficient may not be as difficult as portrayed in Fig. 2.

b) <u>Effect on lift curve</u>

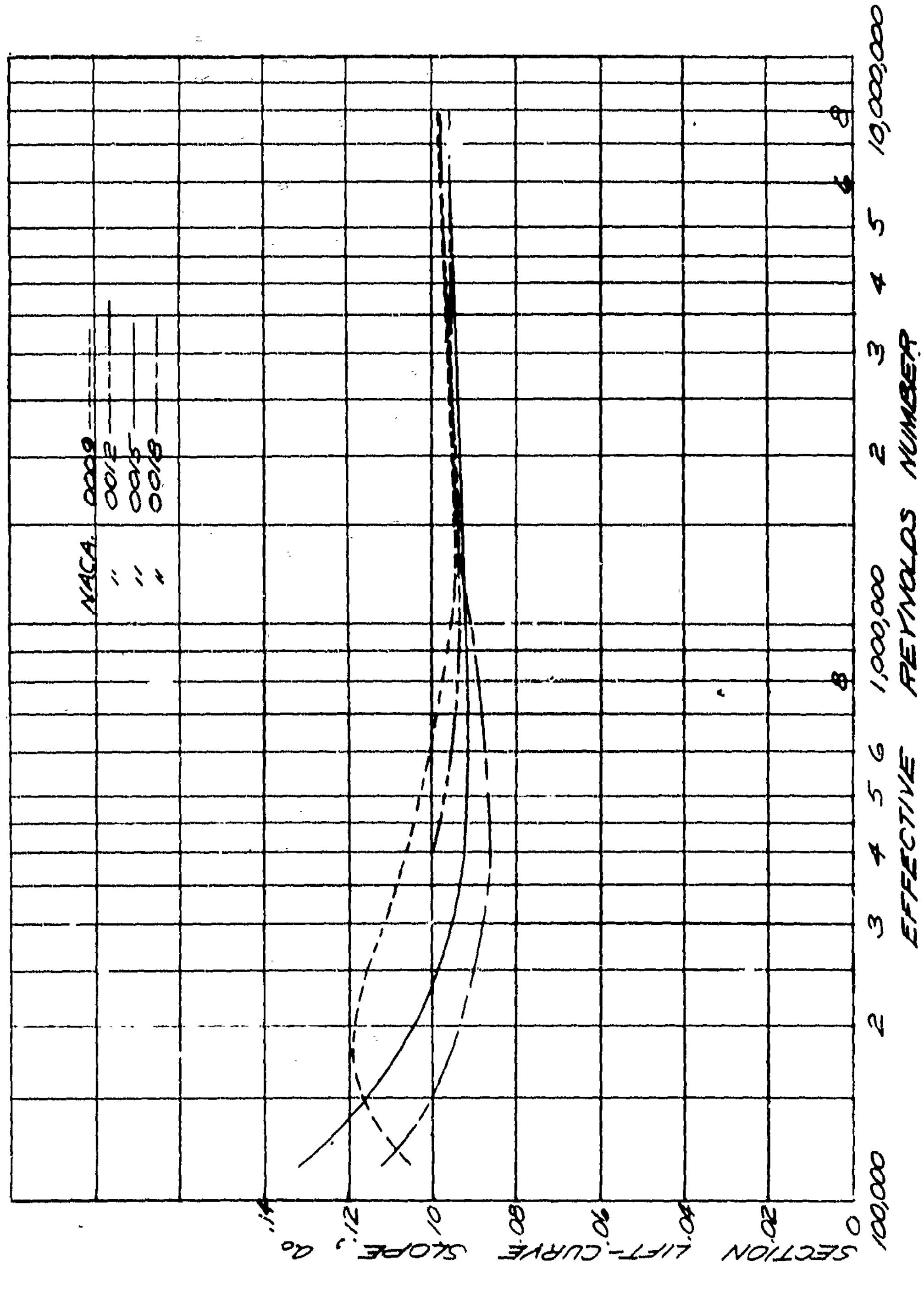
The effect of Reynolds number on the lift is also significant. The

slope of the lift curve decreases (at small R. N.) as R. N. is increased and then increases slightly at the higher R. N.'s (Fig. 4). The maximum lift coefficient and the angle at which it occurs are increased with increasing R.N.'s (Fig. 5).

Mach No Effects

For speeds up to around 300 mph, the condition of similar flow patterns may be met by equal Reynolds numbers. As the speed increases above this region, Mach number effects become increasingly larger, until, as the speed of sound is approached, R. N. effects become secondary as criteria of flow similitude. Unfortunately, whereas an increasing R. N. is usually associated with a decreasing drag coefficient, high Mach number is accompanied by a greatly increased drag coefficient (Fig. 6).

In the subsonic range the lift and lift curve slope increases approximately as $\frac{1}{\sqrt{1-M^2}}$ for a constant angle of attack and the drag coefficient increases to a lesser degree. The drag increase, it is interesting to note, consists largely of increased form drag, the skin friction increasing but slightly.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EFFECTIVE REYNOLDS NUMBER
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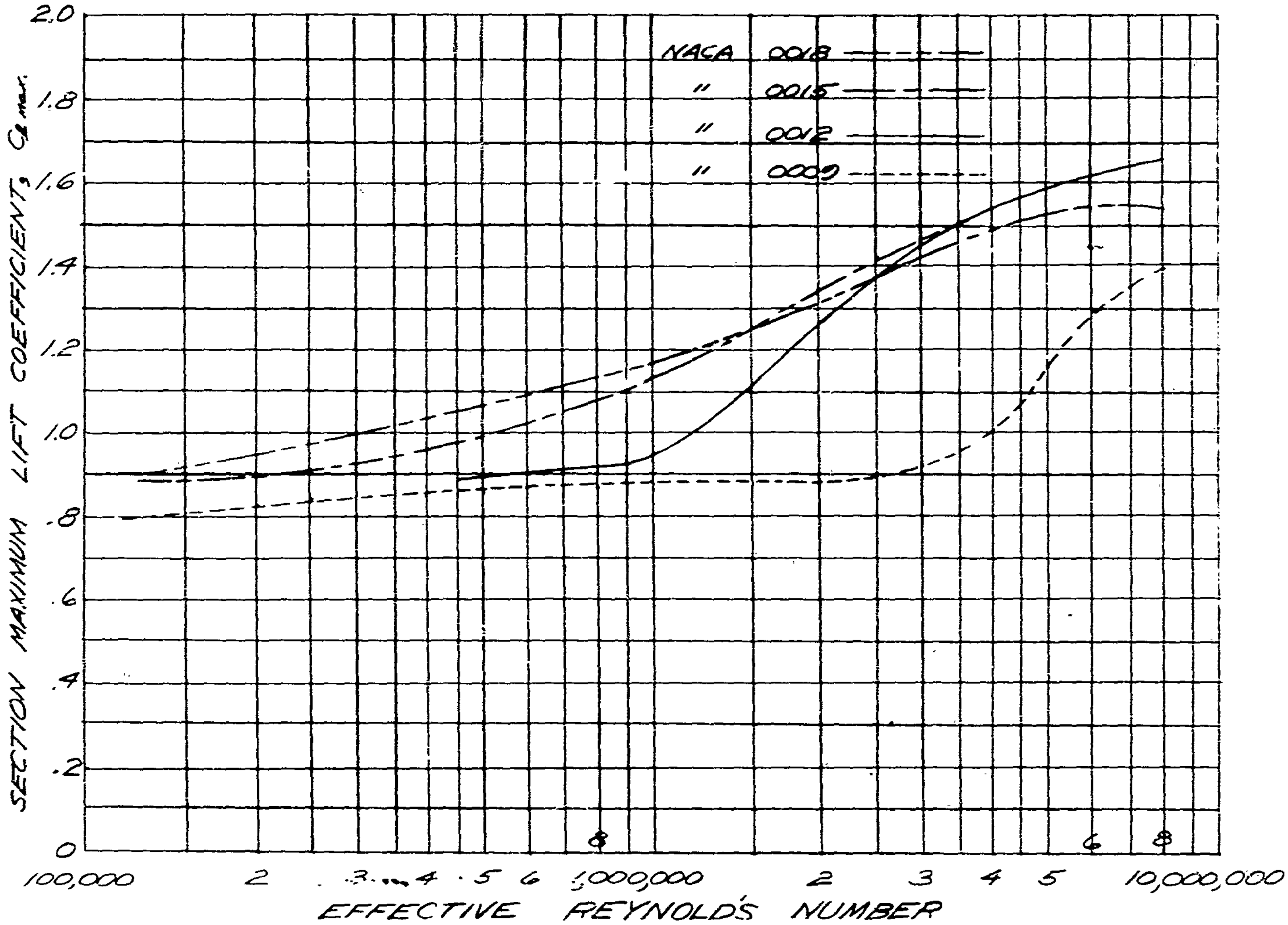
SECTION LIFT-CURVE SLOPE, a_0

FIG. 4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SECTION MAXIMUM LIFT COEFFICIENT, $C_{l\,max.}$
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FIG. 5

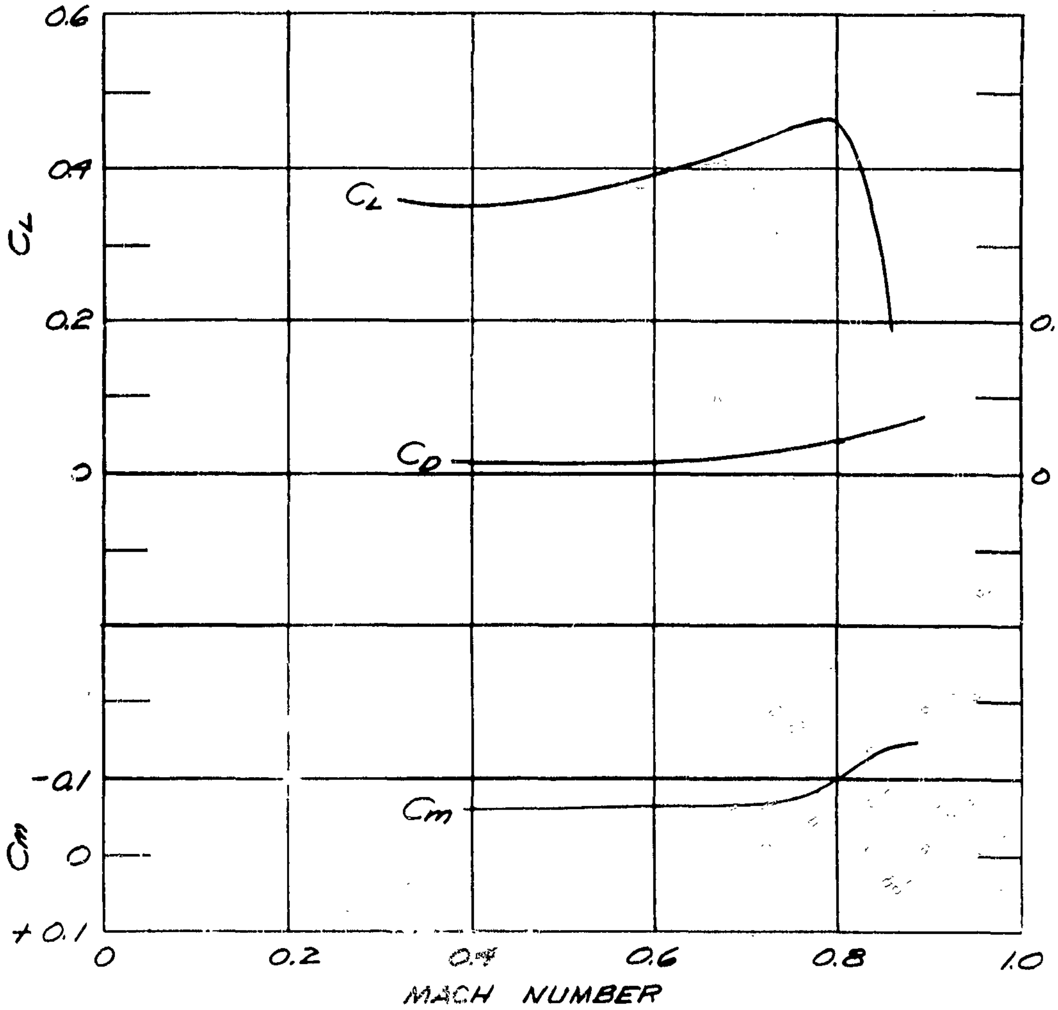

FIG 6

C_L, C_D AND C_m VS. MACH No. AT CONSTANT ANGLE OF ATTACK

5. Conclusions

An attempt has been made to present some of the problems, usefulness and limitations of dynamic model testing as it pertains to the helicopter. Reduction factors for design of dynamically similar models have been presented based on dimensional analysis. A model designed as specified would be expected to exhibit stability and control characteristics similar to the prototype as well as similar rotor blade vibrations, deflections and flutter characteristics.

It has been shown that while the mechanical aspects of simulation can be satisfied in a rigorous manner, the aerodynamical aspects are relatively difficult. In fact, the inability to maintain a reasonable value of model Reynold's number introduces the greatest problem to the development of this useful testing technique. However, the problem although requiring assiduous effort is not insurmountable.

The development of this testing technique would naturally provide the helicopter designer with a powerful tool with which he can economically evaluate, modify and perfect his designs. The effort required for the development of this technique therefore appears warranted.

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